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# A SIMPLE PHYSICAL MODEL TO STUDY THE RELATIONSHIP BETWEEN THE INDIVIDUAL AUTOMOBILE PROPERTIES AND THE COLLECTIVE TRAFFIC BEHAVIOR

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***Abstract.*** *The physical parameters of automobiles (e.g, mass and drag coefficient) are important factors that influence their acceleration and energy consumption. However, automobiles do not travel alone. Their behavior also depends on the traffic conditions. Traffic is a complex system composed by many vehicles that interact at any given instant in a way the individual properties strongly influence the collective behavior. In general, there is a limitation in the way the studies are performed. Since the engine, vehicle, and traffic are compartmentalized in their areas of study, the correlation between the individual cars and the collective traffic is not fully analyzed. In this study, it is proposed a simple traffic model that takes into account the mass, aerodynamic drag, engine power, maximum speed allowed by law, strategies to avoid a collision, and the drivers wish to accelerate. The scenario of simulation is a single-lane oval track, where different number of vehicles are placed. The results of the simulation with different traffic conditions reproduce the fundamental relationship between density, speed, and traffic flow with a triangular shape. Furthermore, the densities of vehicles on the track are also related with restriction in traveling speed. In the free traffic state, the speed limit of the track (defined by law) is the main cause of speed restriction, while in congested traffic the safety is the major responsible for vehicle behavior. Besides, doubling the mass and maximum acceleration desired by the driver results in a difference of 1.7% and 3.1% in the maximum traffic flow, while increasing the maximum deceleration desired by the driver decreased the maximum traffic flow by 55%. However, all parametric studies displayed meaningful differences in the energy analysis.*

***Keywords:*** *complex system, traffic model, fuel consumption, fuel economy, engine model*

## 1. INTRODUCTION

The scientific division of knowledge is not always a good idea. Let's take as example the subject of this paper: vehicles in movement. How this physical phenomenon is studied? The engine performance is studied by Thermodynamics, the motion of the vehicle itself by Dynamics, and the interaction between vehicles by Traffic Science. The problem is all those equipment/phenomena are closely interrelated and should not be compartmentalized. The acceleration of the vehicle, for example, depends on the power of the engine, on the resistances to the movement, on the will of the driver and on the space available on the street.

The traffic of motor vehicles is a complex phenomenon that is sensitive to the number of vehicles, drivers behavior and vehicles characteristics. Traffic modeling allows the characterization and investigation of the influence of parameters such as speed and composition of the fleet on the average behavior of the vehicles on the road (Hodas and Jagota, 2003). Currently there are many models that can be classified according to the scale (microscopic, macroscopic or mesoscopic), time (discrete or continuous) and space/speed (discrete or continuous) (van Wageningen-Kessels *et al.*, 2015). A microscopic model is used in this study, *i.e.*, each car is modeled individually (the complex behavior observed on a macroscopic scale is result of their interactions). There are others classifications in the literature for microscopic models, such as car-following (Wiedemann, 1974; Gipps, 1981; Treiber *et al.*, 2000) and cellular automata (Nagel and Schreckenberg, 1992; Kerner *et al.*, 2002; Meng *et al.*, 2007).

Usually, the microscopic traffic models do not take into account explicitly the influence of engine and physical characteristics, but these models are able to describe the traffic qualitatively and quantitatively when real data is used to calibrate the input parameters (Treiber and Kesting, 2013). However, the vehicle characteristics (*e.g.*, mass, aerodynamic, engine curve) influence in the vehicle acceleration. It is widely known the decreasing relationship between acceleration and speed, as observed by Long (2000) in his literature review and by Fadhloun *et al.* (2015) by using a mathematical model that considers the resistances to move (inertia, drag, rolling). The integration of a traffic model and a vehicular model is a way to take into the account the vehicle characteristics explicitly. Rakha *et al.* (2012) proposed an integrated model that uses the models developed by Ni and Henclewood (2008) to represent the engine power curve; and with a similar model Santos (2019) observed a high occurrence of unreal acceleration in traffic models in which the engine is not modeled. Differently, in VISSIM (PTV, 2018), a traffic simulation software widely used for academic and commercial purposes, the user can define the maximum acceleration for each speed (an approach used to specify implicitly how the engine influences the vehicle movement).

In this paper, a simple traffic model is proposed using basic concepts of physics to describe vehicle motion to study the relationship between vehicular and collective behavior under different traffic conditions. The proposed model was developed in order to be used in undergraduate and graduate courses to study the behavior of vehicles in the traffic accounting the effects of resistances to movement (*i.e.*, aerodynamic drag, tire deformation and gravity), driver (maximum acceleration and deceleration) and safety. Using this model, it is possible to evaluate the influence of the maximum speed of the track, maximum acceleration of the driver, mass of the vehicle, aerodynamic resistance and maximum engine power in the average behavior of the vehicles in traffic condition.

## 2. Methodology

The proposed model can be divided into the modeling of vehicle dynamics, traffic interactions, and consumption calculation. A summary of quantities used in the model are displayed on Table 1.

Table 1. Summary of quantities used in the model.

| Quantity | Definition | Equation | Unity |
|---|---|---|---|
| $t$; $t_0$; $t_a$; $\Delta t$ | Time simulated; Initial period required for system stabilization; Period analyzed; Reaction time and update time | N/A | s |
| $V_i(t)$ | Actual speed of $i^{th}$ car | Eq. (7) | m/s |
| $x_i(t)$ | position of $i^{th}$ car | N/A | m |
| $F_p$ | The maximal traction force available. Where $P_{\text{eng}}$ is the maximum mechanical power available by the engine at given rpm and $\eta_{trans}$ is the transmission efficiency | $\frac{P_{\text{eng}}\eta_{\text{trans}}}{V_i(t)}$ | N |
| $F_a$ | The drag resistance. Where : $k_a$ is the drag coefficient and *W* is the speed of wind | $k_a[V_i(t)-W]^2$ | N |
| $F_{g,x}$ | The gravitational force in the direction of the movement. Where: $m_c$ is the mass of vehicle and driver, *g* is the gravity and $\theta$ is the slope on the road | $m_c g \sin\theta$ | N |
| $F_r$ | The rolling resistance. Where : $C_r$ is the rolling coefficient | $C_r m_c g \cos\theta$ | N |
| $a_i^p(t)$ | Maximum acceleration available for $i^{th}$ car | Eq. (2) | m/$s^2$ |
| $a^m$ | Maximum acceleration desired by the driver | N/A | m/$s^2$ |
| $b$ | Maximum deceleration desired by the driver | N/A | m/$s^2$ |
| $L_v$; $L_t$ | Vehicle length; and Oval track Length | N/A | m |
| $\Delta s_i$ | Distance required by $i^{th}$ car to stop if uses *b* when is initially traveling at $V_i^s$(hypothetical new speed) | $\frac{-[V_i^s(t+\Delta t)]^2}{2b}+\frac{[V_i(t)+V_i^s(t+\Delta t)]\Delta t}{2}$ | m |
| $\Delta s_{i-1}$ | Distance required by the car in front of $i^{th}$ car to stop if uses *b* when is initially traveling at $V_{i-1}$ | $\frac{-[V_{i-1}]^2}{2b}$ | m |
| $D_i$ | Distance between rear bumper of leader vehicle and front bumper of $i^{th}$ car. Where: *n* is the number of cars on the track | $\begin{cases} x_n - x_i - L_v - L_t & \text{, for } i=1\,, \\ x_{i-1} - x_i - L_v & \text{, for } i>1\,. \end{cases}$ | m |
| $V_i^p$ | Maximum speed that vehicle could reach using $a_i^p$ | Eq. (3) | m/s |
| $V_i^s$ | Safety speed (Maximum value to avoid collision) | Eq. (5) | m/s |
| $V^l$ | Maximum speed allowed on the track | N/A | m/s |
| $V_i^a$ | Maximum speed that driver is willing to reach using $a^m$ | Eq. (6) | m/s |
| $P_i^m(t)$ | Mechanical power used by $i^{th}$ car | Eq. (9) | W |
| $C_i(t)$ | Fuel consumption of $i^{th}$ car | Eq. (10) | l |
| $\rho$ | Density of cars in track. Where:*n* is the number of cars in track and $L_t$ is in km | $\rho = n/L_t$ | cars/km |
| $\bar{V}_j(t)$ | Mean speed of *n* cars at given *t* | $\frac{1}{n}\sum_{i=1}^{n} V_i(t)$ | m/s |
| $\bar{\bar{V}}_j$ | Mean speed of *n* cars in track over $t_a$ | $\frac{1}{t_a}\sum_{t=t_0+1}^{t_0+t_a} \bar{V}_j(t)$ | m/s |
| $\bar{\bar{\bar{V}}}$ | Mean speed of *n* cars in track over $t_a$ and *r* (number of repetitions) | $\frac{1}{r}\sum_{j=1}^{r} \bar{\bar{V}}_j$ | m/s |
| $Q$ | Traffic flow in track using mean speed($\bar{\bar{V}}_j$ or $\bar{\bar{\bar{V}}}$). $\bar{\bar{Q}}_j$ for the traffic flow of each repetition (*j*) and $\bar{\bar{\bar{Q}}}$ for average traffic flow over *r* (number of repetitions) | $\bar{\bar{\bar{Q}}} = \bar{\bar{\bar{V}}}\rho$ or $\bar{\bar{Q}}_j = \bar{\bar{V}}_j\rho$ | cars/h |
| $\sigma$ | Mean standard deviation of speed of *n* cars in track over $t_a$and *r* (number of repetitions) | $\frac{1}{r}\sum_{j=1}^{r}\{\frac{1}{t_a}\sum_{t=t_0+1}^{t_0+t_a}[\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(V_i(t)-\bar{V}_j(t)^2}]\}$ | m/s |
| $E$ | Mean fuel economy of *n* cars in track over $t_a$ and *r* (number of repetitions). Where the distance traveled is in km: $\Delta x_i = x_i(t+\Delta t) - x_i(t)$ | $\frac{\frac{1}{r}\sum_{j=1}^{r}\{\frac{1}{t_a}\sum_{t=t_0+1}^{t_0+t_a}[\frac{1}{n}\sum_{i=1}^{n}\Delta x_i]\}}{\frac{1}{r}\sum_{j=1}^{r}\{\frac{1}{t_a}\sum_{t=t_0+1}^{t_0+t_a}[\frac{1}{n}\sum_{i=1}^{n}C_i(t)]\}}$ | km/l |

### 2.1 Vehicle sub-model

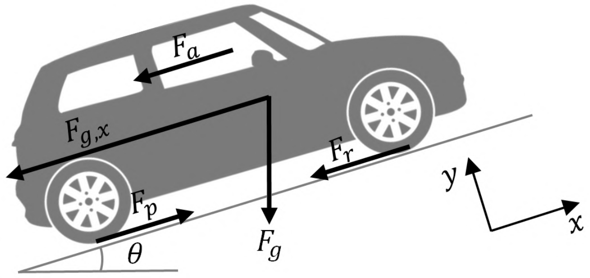


Figure 1. Balance of forces acting on the vehicle

The vehicle sub-model, Eq. (1), defines the maximum physical acceleration the $i^{th}$ vehicle could travel accounting the forces related to the movement, *i.e.*, drag resistance ($F_a$), rolling resistance ($F_r$), gravitational resistance in the direction of the movement ($F_{g,x}$), and the maximum traction force that could be provided by the engine ($F_p$). The term on the left-hand side of the equation 1 is the inertia, where $m_c$ is the combined mass of vehicle and driver, *t* is time and V is the vehicle speed. The mathematical definitions of these forces are displayed on Table 1.

$$m_c \frac{\Delta V}{\Delta t} = m_c a^p = F_p - F_a - F_{g,x} - F_r \tag{1}$$

Substituting the forces (see Table 1) in Eq. (1), the maximum acceleration of the $i^{th}$ vehicle could physically reach for each speed can be calculated :

$$a_i^p(t) = \frac{P_{\text{eng}}\eta_{\text{trans}}[V_i(t)]^{-1} - k_a[V_i(t) - W]^2 - m_c g \sin\theta - C_r m_c g \cos\theta}{m_c} \tag{2}$$

In this paper, the engine power curve ($P_{\text{eng}} = f(\text{engine speed [rpm]})$) is the one given by Ni and Henclewood (2008), calculated to respect the maximal power of the engine ($P_{\text{max}}$), the engine speed of the maximal power and the engine speed of the maximal torque — all information available in car manuals. In Eq. (3) it is calculated the maximum speed that the $i^{th}$ vehicle could reach after a time interval ($\Delta t$) using $a_i^p(t)$.

$$V_i^p(t + \Delta t) = V_i(t) + a_i^p(t)\Delta t \tag{3}$$

### 2.2 Traffic sub-model

The traffic sub-model is used to calculate the maximum speed that a vehicle (plate *i*) could travel without collision, $V_i^s$. The safety speed is calculated taking into account the distance of two near vehicles (the leader with plate *i-1*, and the follower with plate *i*) would travel if both applied the brakes using the maximum desired deceleration (*b*). It is also considered the follower vehicle (plate *i*) would take a time ($\Delta t$) to react, and the driver always wants to keep a minimum distance ($D_{min}$) to the leader vehicle, Figure 2. This approach is similar to the model proposed by Gipps (1981).

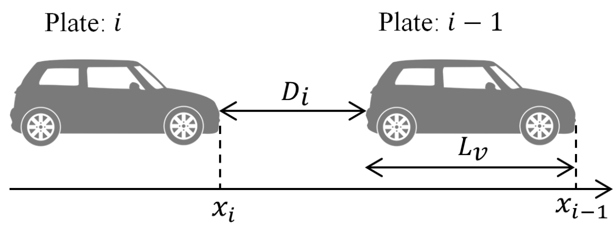


Figure 2. Follower vehicle and leader vehicle.

Eq. (4) is the condition that must be met in order to not result in a collision between the two vehicles. The first two terms in the left are the distance the leader and follower vehicle would travel in order to stop completely. The first term of both $\Delta s_i$ and $\Delta s_{i-1}$ derive from Torricellis equation, and the second term of $\Delta s_i$ accounts the delay of follower car to react to the braking of the leader (see Table 1). $L_v$ is the vehicles length and $D_i$ is the actual distance between both vehicles. In this model it is considered that all vehicles are traveling on an oval track of total length represented by $L_t$.

Thus, considering there are *n* vehicles on the track, the vehicle in the front of the pack (plate 1) will always see in front of it the vehicle at the end of the pack (plate *n*).

$$\Delta s_{i-1} - \Delta s_i - D_i \geq D_{\text{min}} \tag{4}$$

The safety speed is calculated using Eq. (5), obtained by manipulation of Eq. (4) and considering the limit case (equality sign). The Eq. (5) is the solution of a second-degree polynomial equation in $V_i^s$ .

$$V_i^s(t+\Delta t) = \frac{b\Delta t}{2} + \sqrt{(\frac{-b\Delta t}{2})^2 + [V_{i-1}(t)]^2 - 2b(D_i - D_{\text{min}}) + bV_i(t)\Delta t} \tag{5}$$

This approach will result in a speed reduction in next time step if the actual speed not ensure collision avoidance if the leader vehicle start to brakes with *b* for given distance between vehicles, as shown in Figure 3. In this example, the leader vehicle is slower and is braking with *b* (both vehicles reach stationary state after 6 second with approximately $D_{\text{min}}$).

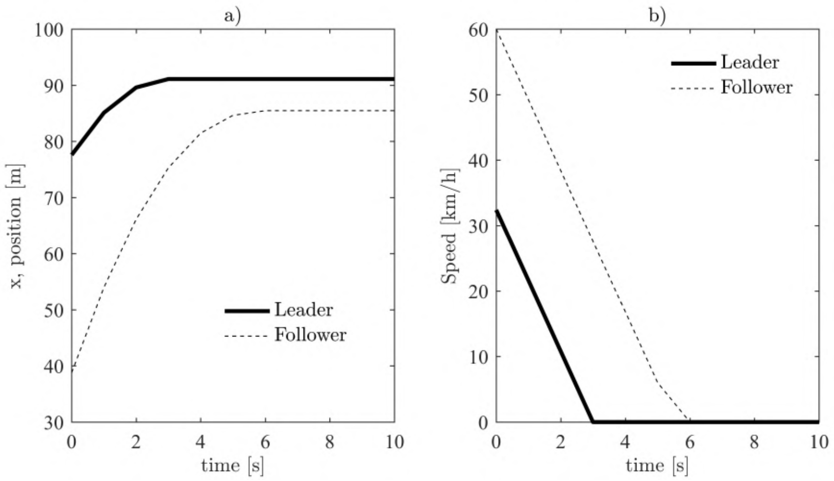

Figure 3. Evolution of a)position and b)speed of two cars under safety criteria.

The updated speed of the vehicle plate *i* is determined considering the limitations of the engine/vehicle ($V_i^p$, Eq. (3)), law ($V^l$, an input parameter), safety ($V_i^s$, Eq. (5)), and driver's wish ($V_i^a$, Eq. (6)).The driver's wish is represented by the speed the vehicle could reach after a time interval using the maximum desired acceleration ($a^m$, an input parameter).

$$V_i^a(t+\Delta t) = V_i(t) + a^m \Delta t \tag{6}$$

Furthermore, during the simulation there is a probability *p* for each vehicle to apply the brake with a deceleration *b*. This probability is an important parameter to represent driving imperfections in traffic due human behavior. Without this parameters all vehicles in track would reach stationary state quickly (Nagel and Schreckenberg, 1992), as if wagons in a train. The determination of which vehicle will brake randomly is made by comparison between *p* and a number $q_i(t)$ generated randomly for each car in each time step. Finally, the vehicle speed is updated using Eq. (7):

$$V_i(t+\Delta t) = \begin{cases} \min\left\{V_i^p(t+\Delta t), V_i^a(t+\Delta t), V_i^s(t+\Delta t), V^l\right\} & \text{, for } q_i(t) \geq p\,, \\ \max\left\{0, V_i(t) + b\Delta t\right\} & \text{, for } q_i(t) < p\,. \end{cases} \tag{7}$$

The calculations that until now culminating in equation Eq. (7) are the foundation stone of vehicles motion. It is also important to point out that in the proposed model the vehicles only travel in a straight line. The representation of the movement on curves and under different weather or road conditions would require a more detailed analyses of forces acting on the vehicles during braking (Magnani and Cunha, 2017).

### 2.3 Consumption sub-model

The new speed calculated with the vehicle and traffic sub model allows the calculation of the instantaneous acceleration of each vehicle on the track, Eq. (8).

$$a_i(t) = \frac{V_i(t + \Delta t) - V_i(t)}{\Delta t} \tag{8}$$

The mechanical power needed by the car in each moment is given by Eq. (9), where $\eta_{\text{trans}}$ is the transmission efficiency and $P_{\text{idle}}$ is the engine mechanical power necessary when the car is stopped (internal friction and accessories). The numerator is the summation of the resistance forces (inertia, drag, rolling and gravity) multiplied by the speed.

$$P_i^m(t) = \frac{(m_c a_i(t) + F_a + F_r + F_{g,x} + F_a)V_i(t)}{\eta_{\text{trans}}} + P_{\text{idle}} \tag{9}$$

The result of Eq. (9) is the mechanical power required from the engine to accelerate the vehicle. In order to calculate the amount of fuel used it is necessary to take into account the energy losses during combustion (engine efficiency) and also the quantity of heat the fuel will release during combustion (an experimental quantity called heating value). The Eq. (10) is used to calculate the instantaneous consumption, where $\eta_{eng}$ is the engine efficiency, $\rho_{fuel}$ the fuel density, and $H$ its heating value.

$$C_i(t) = \max\left(0, \frac{P_i^m(t)}{\eta_{\text{eng}}\rho_{\text{fuel}}H}\right) \tag{10}$$

The model considers the fuel consumption being null during deceleration when the engine power is not required (Andrade *et al.*, 2021), *i.e.*, in the situations where the throttle pedal is not being used ($P_i^m(t) < 0$), Eq. (10). In those situations it is considered the vehicle is travelling due to inertia alone.

### 2.4 Simulation

Each simulation is performed for 1500 seconds on the 2.25 km ($L_t$) track, Figure 4 . The first 500 seconds ($t_0$) are discarded as the time to the system to became stable, thus the averages (*e.g.*, traffic flow, average consumption) are for the last 1000 seconds ($t_a$). Additionally to the initial discard, for each density of vehicles, 50 ($r$) repetitions are made in order to decrease the effects of the randomness, both from the initial condition (in the beginning of each simulation, all cars are stopped and allocated in random positions) and from the random braking (Eq. (7)), when $q_i$(t)< $p$). Each set of 50 repetitions has a particular number of vehicles ($n$) on the track. The density of vehicles on the track varies in the range between 10 and 140 vehicles/km.

The update time for the vehicles and the reaction time of the driver are defined with the same value, 1s ($\Delta t$). It is important to highlight that $V_i^p(t{+}\Delta t)$ is obtained using Finite Difference Method in its forward form (Eq. (3)), so there would be a considerable error in low speed for this value of $\Delta t$. For example, a stationary vehicle could reach any speed desired, because the maximal traction force would be infinite ($F_p = P_{\text{eng}}\eta_{\text{trans}}/[V_i(t)]$) if the speed was null. However, the limitation of maximum acceleration (the input parameter $a^m$) prevents that unrealistic values speed. This error could be minimized with a lower update time, but that would result in a longer computation time for simulation.

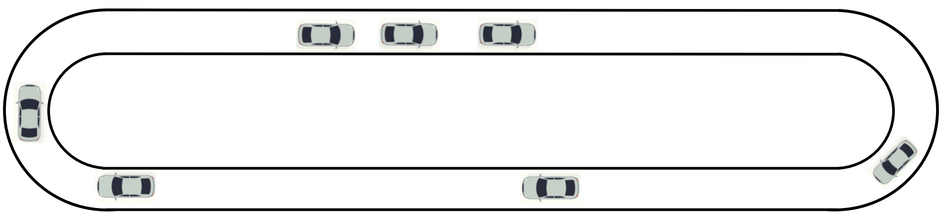

Figure 4. Single-lane track used for simulation.

## 3. Results and discussion

The traffic sub-model integrated with the vehicle sub-model allows to evaluate the influence of parameters related to the track, driver and vehicle in the fuel consumption and traffic conditions. Previously, Magnani *et al.* (2018) used a simple vehicular model to evaluate the influence of engine and physical characteristics of the vehicle in the performance of motorcycles. However, in that model the motorcycles were not placed in traffic conditions. When the vehicles are traveling with others on the road they may be subject to deceleration and speed limitations to avoid collisions that would not happen if they traveled alone.

In Figure 5 is shown the behavior of one car in track during 250 seconds. This kind of figure is called trajectory diagram, where the line show how the vehicle travel in time. In this case, the car is alone on the track and can travel with maximum speed of 60 km/h and $p$ is the probability of randomly decelerate with -0.7 m/$s^2$ in each time step. The dotted line, with higher braking probability, has a lower slope if compared with the other line. This indicates that the vehicle is traveling slower in average than the vehicle in the other simulation. However, there is not a meaningful qualitative difference between the behaviors of both vehicles.

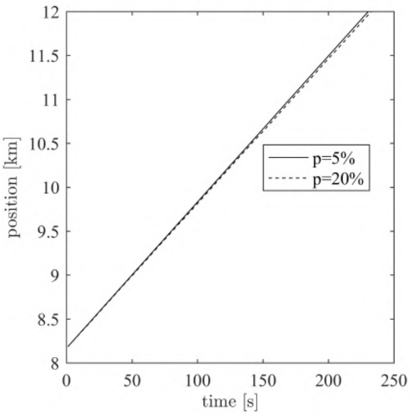

Figure 5. Trajectories diagram of a single vehicle on the track with different braking probabilities.

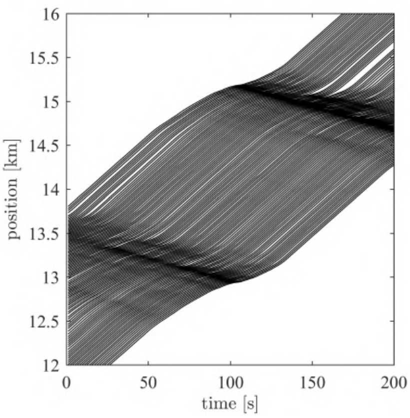

Figure 6. Trajectories diagram of 112 vehicles on the track with $p = 5\%$.

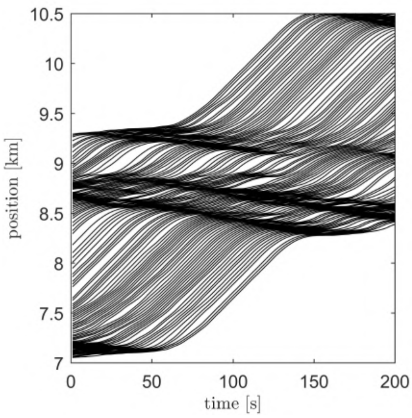

Figure 7. Trajectories diagram of 112 vehicles on the track with $p = 20\%$.

Contrariwise, when there are more vehicles traveling together, the traffic is more sensitive to the drivers behavior. In Figure 6 it is show the movement of 112 vehicles for $p$=5%. It can be seen that there are regions where the vehicles are further away of each other (blank spaces) and also that the behavior of the leader vehicles influence the follower vehicles. The almost horizontal dark regions represent traffic congestion. In those regions one vehicle traveling slowly or remaining in stationary state will result in the deceleration of follower vehicles. This is a real traffic phenomenon that amazes anyone, in which the traffic suddenly slows without a perceptive reason only to free itself again a little latter. This kind of highly non-linear phenomenon is a common feature of complex systems (*e.g.*, traffic constituted by simple vehicles, a colony constituted by simple ants, a brain constituted by very simple neurons) (Boccara, 2010). Complex systems show complex behaviors that could not be anticipated by examining the isolated behavior of its parts (Figure 5 to Figure 7).

The traffic behavior on track is completely different for the same number of vehicles but now with 20% of braking probability, Figure 7. This difference was very small with the vehicles alone (Figure 5). In this new condition, the vehicles remain more time traveling slowly and there is a greater occurrence of accelerations and braking. This means, in addition to a lower flow on the track, a higher fuel consumption. These examples are important to understand how the consideration of traffic condition is important to analyze the vehicular performance.

### 3.1 Base Case

The Base Case used for the initial analyses and further comparisons uses the parameters displayed in Table 2.

Table 2. Parameters of Base Case.

| Vehicle | Track and driver | Consumption |
|---|---|---|
| $P_{\text{max}}$ = 56.6×$10^3$W | $\theta$= 0° | $\eta$ = 0.20 |
| $k_a$ = 0.4 kg/m | $D_{\text{min}}$ = 2 m | $P_{\text{idle}}$ = 1.1×$10^3$W |
| $C_r$ = 0.01 | $V^l$ = 16.7 m/s | Fuel=E22 |
| $m_c$ = 1076 kg | $a_m$ = 1.0 m/$s^2$ | H = 38.9×$10^6$J/kg |
| $\eta_{\text{trans}}$ = 0.95 | $b$ = -0.7 m/$s^2$ | $\rho_{\text{fuel}}$ = 0.745 kg/l |
| $L_v$ = 4 m | $p = 5\%$ | - |

In traffic science there is a relation that is widely used to represent traffic flow graphically that is called the Fundamental Diagram. This diagram is used to show the relation of traffic flow (or average speed) and vehicle density. In the diagram flow-density (Figure 8) there are two well-defined regions: free traffic phase and congested traffic phase. The free traffic phase is characterized by an increasing flow with the number of vehicles on the track, up to $\rho = 43$ (critical density). The maximum traffic flow ($\bar{\bar{Q}}_j$) observed is 2395 car/h , which represents the end of the free traffic phase and the beginning of the congested phase.

As an example, the car flow for $\rho = 50$ represent the average behavior on track when there are 112 vehicles and

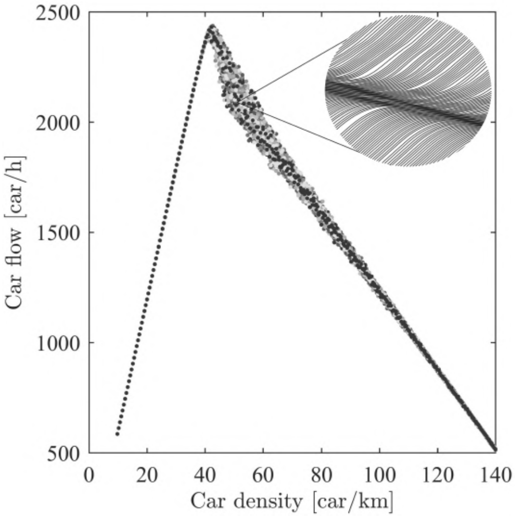


Figure 8. Diagram of flow-density of Base Case ($\bar{\bar{Q}}_j$).

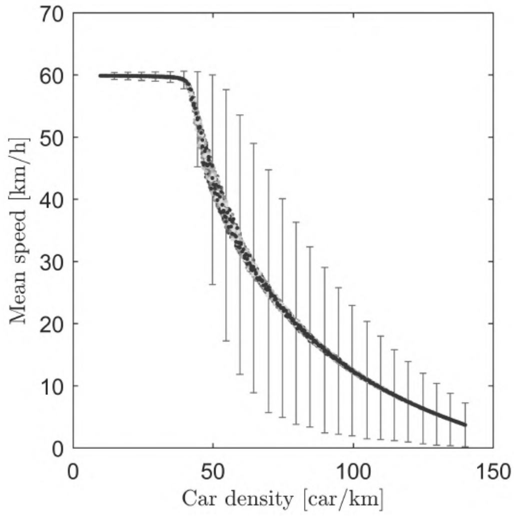


Figure 9. Diagram of speed-density (with speed standard deviation).

*p*=5%, exactly the example shown in previous section (Figure 6). In other words, each point in the Fundamental Diagram (Figure 8) is the average behavior observed on the track for a traffic condition — as seen in the trajectories diagram.

In Figure 9 it is observed that in the free traffic phase the vehicles are traveling with values close to the maximum allowed speed (60 km/h). However, there is a meaningful decrease in speed after the critical density. In Figure 9 it is also displayed the standard deviation ($\sigma$) of the speed during simulation. This quantity is useful because it is related to the magnitude and occurrence of accelerations in track. A low value of $\sigma$ indicates that all vehicles are traveling with similar speed (high speed for the free phase and slow speed for the rightmost part of the congested phase) and that there is low occurrence of acceleration, as one would expect can expect for small densities (the vehicles do not disturb each other) and also for large densities (all the vehicles are nearly stopped). On the other hand, high $\sigma$ indicates that the vehicles are traveling with different speeds and the vehicles are accelerating and braking more.

The average fuel economy on track varies from 17.2 to 5.5 km/l, Figure 10. One vehicle traveling alone in track with the same input parameters of the Base Case would have a very similar fuel economy (17.4 km/l) and average speed (59.9 km/h) as in the traffic with few vehicles (*e.g.*, $\rho$ = 10).

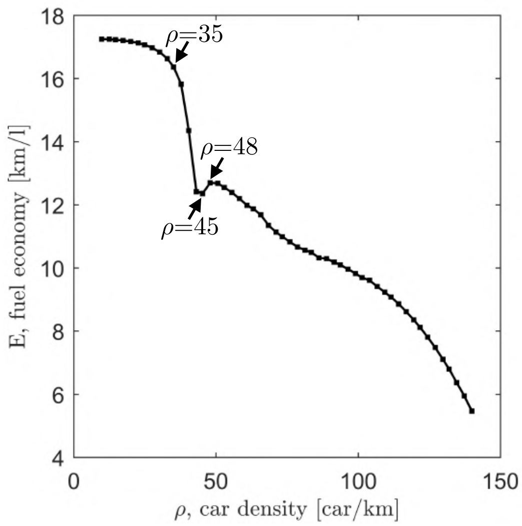


Figure 10. Fuel economy of the Base Case.

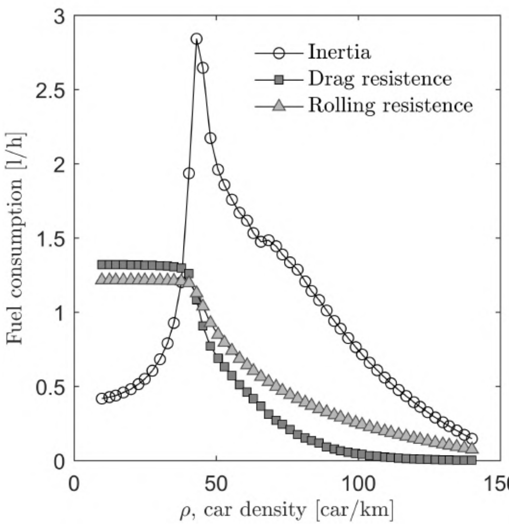


Figure 11. Fuel consumption due resistive forces.

The difference of fuel economy in each car density (Figure 10) can be analyzed in term of resistive forces, Figure 11. The consumption caused by the rolling resistance ($P_r = F_r V = C_r m_c g \cos\theta V$) varies linearly with the speed, remembering that $C \propto P = FV$ (*i.e.*, the consumption is proportional to the force times speed). The power required to overcomes the aerodynamics resistance ($P_a = F_a V = k_a V[V - W]^2$) varies in a cubic manner, and the resistance caused by the inertia is related to the acceleration and mass. The fuel economy curve shows a decreasing relationship with the car density, except from 45 to 48 cars/km. The decrease of speed not necessarily means a decrease of the fuel consumed, because i) the occurrence of re-accelerations can (almost always does) result in higher consumption due to the inertial resistance and ii) the cars that are stopped in the traffic jam burn fuel (idle) even without moving.

The vehicles have distinct behavior under different traffic conditions, which explain the differences in values of energy performance, Figure 10. In the free traffic phase (up to 35 cars/km) the vehicles can travel with higher speed and low occurrence of acceleration, because of that the fuel economy is the highest observed (17.2 km/l for $\rho$ = 10 cars/km). After that density, there are a greater occurrence of acceleration with higher speed at densities between 35 and 45 cars/km, resulting in a higher consumption due to the inertia. In the last part of the curve, above 48 cars/km, the vehicles travel more and more slowly with high densities, decreasing the distance traveled in comparison to fuel consumption. In the congested traffic phase, close to the critical density ($\rho$ = 43 cars/km), the movement is more chaotic because there are regions that allow vehicles to travel with high speed but also congested regions. For example, the small increase of fuel economy between 45 and 48 cars/km is the result of this chaotic section, in which the decrease of fuel consumption is 7.2% higher than the decrease of distance traveled.

The update of speed in each time step is made by taking the minimal of $V_i^p$, $V_i^a$, $V_i^s$, $V^l$, Eq. (7). In each one of those kinds of speed one different aspect in considered: engine/vehicle characteristics, drivers wish, safety, and law. As can be seen in Figure 12, during the free phase the main limitation is performed by $V^l$ (to avoid traffic fines), but after the critical density the safety criteria dominates the decision of the speed (to avoid collision). In this Base Case, the speed was never restrained by the physics of movement. This would not be the case if the engine was weaker, the allowed speed was higher, or if the driver was eager to greater accelerations.

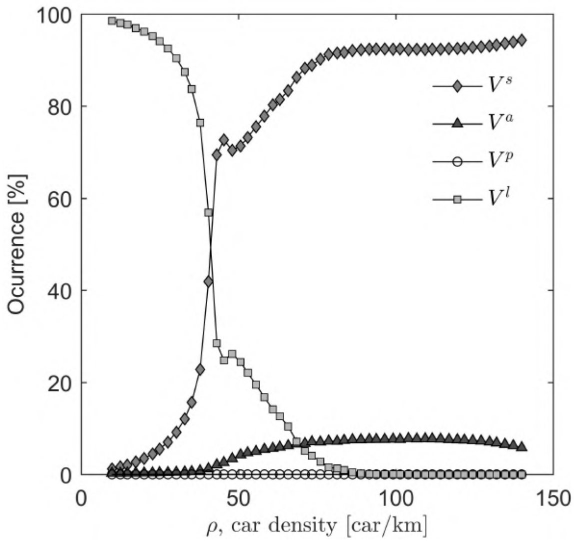


Figure 12. Speed type occurrence for different traffic conditions.

### 3.2 Parametric study

The parameters that were modified to study the influence of the driver/vehicle are shown in Table 3. As expected, a higher mass (Case A) results in a larger fuel consumption; a higher maximal deceleration (Case C) decreases traffic flow and fuel economy; and a lower braking probability (Case D) increases traffic flow. The impact of a higher acceleration (Case B) in the maximum and minimum values presented in table is negligible in comparison with Base Case, because higher accelerations will occur neither at free nor at extremely congested traffic state.

The Figure 13 shows the difference in traffic behavior in comparison to the Base Case. The case with higher mass (Case A) and Base case exhibit similar traffic flow because the limitation imposed by the engine to accelerate is similar to the restriction of acceleration due driver's wish. The Case B exhibits a slight increase in traffic flow after the critical point, because the drivers are willing to use a higher acceleration (*e.g.*, the average acceleration can reach 1.4 m/$s^2$ at congestion phase). In Case C (greater allowed braking), the traffic exhibits similar behavior in the free traffic phase up to $\rho$ = 25, after this density the relatively low $a^m$ can not compensate anymore the more aggressive braking events. In Case D, the vehicles travel with *p*=0, thus there is no speed fluctuation due human behavior. This case can be seen in two ways. First of all, it is used to conclude that would be possible to have 99 cars traveling always with 60 km/h, in the same way as the isolated vehicle on the track. Also, this case could be used to analyze what would happen if there were only autonomous vehicles on the track. Without driving imperfections, the traffic flow ($\bar{\bar{Q}}_j$) would reach 2620 cars per hour and the end of

Table 3. Influence of track and driver parameters in the behavior of vehicles in traffic.

| Case | Varied parameter | Maximum fuel economy (km/l) | Minimum fuel economy(km/l) | Critical density (car/km) | Maximum traffic flow (car/h) |
|---|---|---|---|---|---|
| Base | - | 17.2 | 5.5 | 43 | 2395 |
| A | $m_c$ = 2570 kg | 14.8 | 4.6 | 40 | 2320 |
| B | $a^m$ = 2.0 m/$s^2$ | 17.2 | 6.4 | 43 | 2435 |
| C | $b$ = -3.0 $\mathrm{m/s^2}$ | 12.5 | 4.6 | 25 | 1316 |
| D | $p$ = 0% | 18.5 | 5.8 | 45 | 2620 |

the free phase would occur at $\rho$ = 45. After the critical density the vehicles decrease their speed to prevent the occurrence of collisions because there is less free space on the track.

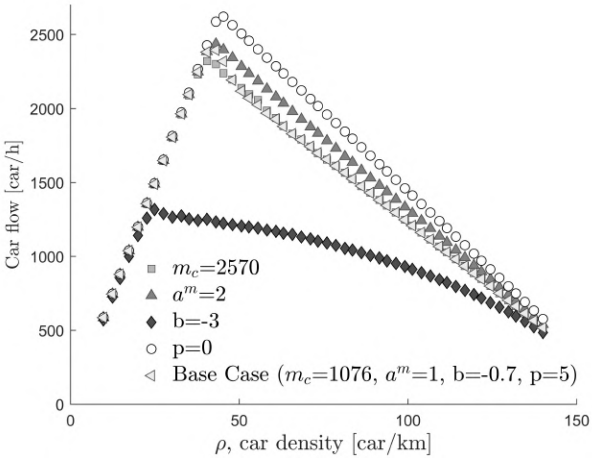


Figure 13. Comparisons between traffic flows ($\bar{\bar{\bar{Q}}}$).

The fuel economy curves vary significantly when the parameters are changed, Figure 14. It should be noticed that the economy is a quantity that depends both on the distance traveled (km) and on the fuel consumption (liters per time). Low fuel consumption per time does not necessarily means a better economy, since the vehicle may be moving slowly. Just as a high consumption can be compensated for by a large distance traveled per time step.

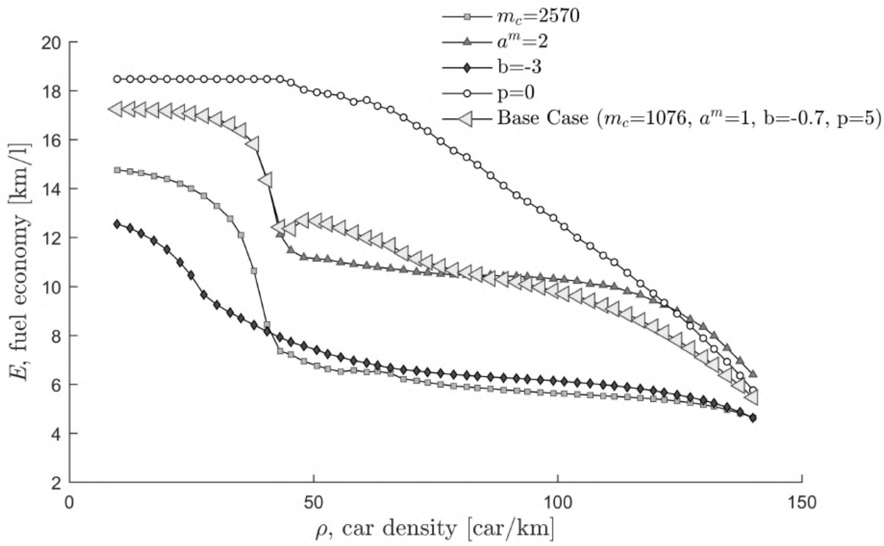


Figure 14. Comparisons between fuel economies.

In Case A the cars use more fuel because of the mass (inertia), that is 1.9 times higher than in the Base Case. In B, the higher $a^m$ results in a increase of fuel consumption also due to inertia. Although it also means that the vehicles have to accelerate for a shorter period of time in the free traffic phase to reach the desired speed. In Case C, there is a more aggressive use of the brakes, which requires more time re-accelerating because of the relatively lower maximum acceleration desired, resulting in lower fuel economy. The Case D exhibited the best energy performance, because there is no occurrence of aleatory deceleration (braking only occurs in the situations to avoid collision, *i.e.*, when there is less free space, after $\rho = 43$) .

## 4. Conclusion

The study of vehicle performance is a field of study that includes concepts from thermodynamics, dynamics, and traffic science. The use of a traffic model integrated with a vehicle model enables us to account the vehicles interactions on traffic for different track densities, a feature that would be ignored when analyzing the vehicular performance of an isolated vehicle. The results showed that fuel economy is a quantity that is not unique, because strongly depends on the number of vehicles on track. Additionally, the aggressiveness of the driver and characteristics of the vehicle also influences in traffic behavior and energy performance. For example, the heavier is the vehicle, the smaller is the fuel economy and achievable acceleration, because of a large inertia resistance.

The input driver parameters, as acceleration and deceleration, have a meaningful impact in traffic behavior that result in large difference in energy performance. The changes observed occurs because the different parameters used may influence the way that vehicles interact on track. The proposed model is simple and captures qualitatively the influence of input parameters on fuel consumption and vehicles' speed.

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